H7110 Project 2 Part 1

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Abstract

This paper presents the design and analysis of a two-stage gearbox transmission, with a Continuously Variable Transmission (CVT) and a fixed gear stage, intended to deliver 5833 W with an input speed of 1000 RPM. Key design parameters include optimising torque, speed, and efficiency while addressing load, pressure, and material durability for both CVT and fixed stages. The study evaluates bearing types and gear configurations. Radial ball bearings are used for the CVT stage and cylindrical roller bearings for the fixed gear stage. Power requirements, force distributions, and Hertzian contact pressures help with material selection. Potential failure modes are analysed.

This list includes the key variables, parameters, and their units used throughout the document for clarity.

Nomenclature

η_{CVT} Efficiency of CVT stage

η_{gear} Efficiency of fixed-gear transmission

η_{total} Overall efficiency of transmission system

ω_{in} Input angular velocity (rad/s)

ω_{out} Angular velocity of output shaft (rad/s)

a Contact radius in Hertzian pressure calculation (mm)

d Diameter (mm)

E' Effective modulus of elasticity for Hertzian contact (Pa)

F_{ball} Force on each ball (N)

i_{fixed} Fixed gear stage ratio

i_{max} Maximum CVT Ratio

i_{min} Minimum CVT Ratio

i_{total} Total effective transmission ratio

N_{in} Input Speed (RPM)

P Output Power (W)

p_{max} Maximum Hertzian contact pressure (GPa)

R_{C1}) Radius of bearing cage (mm)

T_{final} Final output torque (Nm)

T_{in} Input Torque (Nm)

T_{out} Output Torque (Nm)

Contents

1. Declaration

This report is submitted as part requirement for the degree of Mechanical Engineering at the University of Sussex. It is the product of my own labour except where indicated in the text. The report may be freely copied and distributed provided the source is acknowledged. I hereby give permission for a copy of this report to be loaned out to students in future years.

2. Introduction

This project is the first step towards designing a two stage gear box, with a fixed stage and a CVT stage.

2.1. Objectives

(i) To design a gearbox transmission that is in two stages: CVT and fixed gear.
(ii) To provide calculations for the forces, pressures, powers and torques involved to make sensible design decisions.
(iii) To consider materials and modes of failure to ensure that the transmission will not fail.

2.2. Project Description

2.2.1. Parameters

(i) Output power is 5833W
(ii) Input speed is 1000 RPM
(iii) i_{max} CVT is 0.750
(iv) i_{min} CVT is 0.525
(v) Ratio fixed gear stage is 0.698

3. Schematics

The schematic is the last three pages of the document.

4. Bearings

4.1. Rolling Element Bearings type

Rolling element bearing types and their properties are shown in table 1. In this section the bearings supporting rotating parts are looked into in each section of the gearbox.

4.1.1. CVT Stage (Discs and Balls)

Discs and balls have high Hertzian contact stresses because of the EHD traction mechanism. This setup also has rotating parts that need to handle high-speed, low-load conditions and very high efficiency for smooth, continuous speed adjustments. Therefore the choice is radial ball bearings.

They have high efficiency (99%), low friction, and suitability for high-speed rotation with moderate radial loads. Radial ball bearings are common in applications where load is low to moderate, and efficiency is essential.

4.1.2. Fixed Two-Step Gear Stage

The torque and load demands are higher here than those in the CVT. Here, robustness, load-carrying

Bearing Type	Load Capacity	Misalignment Tolerance	Speed Suitability	Application Examples
Ball	Moderate radial and light axial	Low	High	Motors, fans, pumps
Cylindrical Roller	High radial, no axial	Low	High	Gearboxes, heavy-duty conveyors
Spherical Roller	Heavy radial and some axial	High	Moderate	Mining, material handling
Tapered Roller	High radial and axial	Low	Moderate	Automotive, gear drives
Needle Roller	High radial, compact size	Low	Moderate	Transmissions, universal joints
Thrust	High axial only	Low	Low to moderate	Screw jacks, crane hooks

Table 1: Summary of Bearing Types and Their Applications

capacity, and moderate efficiency are more critical than high efficiency or high-speed capability. This stage does not adjust continuously, a high load capacity is more important than speed adjustment.

Therefore cylindrical roller bearings are a good choice. They have high radial load capacity, moderate efficiency (96%), and durability. Cylindrical roller bearings are suited for heavy-load applications where radial forces are dominant, as is often the case in gear stages.

4.2. Diameter of the Balls

Maximum Hertzian pressure in each contact must be within the range 1.2 − 1.5 GPa. The traction coefficient of the oil is 0.045, the average radius of the cage, $RC_1 = 150$mm and there are 6 balls.

4.2.1. Calculate Output Torque

The output power P and input speed (N_{in}) can be used to calculate the torque at the output of the CVT stage.
- $P = 5833\,\text{W}$
- $N_{in} = 1000\,\text{RPM}$

$$P = T_{out} \cdot \omega_{out}, \tag{1a}$$
$$T_{out} = \frac{\omega_{out}}{P}, \tag{1b}$$

- P is the output power in watts, - T_{out} is the output torque in newton-meters (Nm), - ω_{out} is the angular velocity of the output shaft in radians per second (rad/s).

To find ω_{out}, calculate the input angular speed, then adjust it by the gear ratios. $i_{max} = 0.750$ and $i_{min} = 0.525$.

$$\omega_{in} = \frac{1000 \times 2\pi}{60} = \frac{2000\pi}{60} \approx 104.72\text{rad/s} \tag{2a}$$
$$\omega_{\text{out,max}} = \omega_{in} \times i_{\max} \tag{2b}$$
$$104.72 \times 0.750 \approx 78.54\,\text{rad/s} \tag{2c}$$
$$\omega_{\text{out,min}} = \omega_{in} \times i_{\min} \tag{2d}$$
$$104.72 \times 0.525 \approx 54.98\,\text{rad/s} \tag{2e}$$

The fixed gear stage has a ratio of 0.698, so the final output speeds are:

$$\omega_{\text{final,max}} = \omega_{\text{out,max}} \times 0.698 \approx 54.82\,\text{rad/s} \tag{3a}$$
$$\omega_{\text{final,min}} = \omega_{\text{out,min}} \times 0.698 \approx 38.73\,\text{rad/s} \tag{3b}$$

Using $P = T \cdot \omega$, rearrange for torque:

$$T_{\text{final}} = \frac{P}{\omega_{\text{final}}} \tag{4a}$$
$$T_{\text{final,min}} = \frac{5833}{54.82} \approx 106.40\,\text{Nm} \tag{4b}$$
$$T_{\text{final,max}} = \frac{5833}{38.73} \approx 150.61\,\text{Nm} \tag{4c}$$

4.2.2. Force on Each Ball

There are 6 balls, assume the load is evenly distributed, so each ball carries one-sixth of the total torque load.

$$F_{\text{ball,max}} = \frac{T_{\text{final,max}}}{R_{C1} \times 6} = \frac{150.61}{0.15 \times 6} \approx 167.34\,\text{N} \tag{5a}$$
$$F_{\text{ball,min}} = \frac{T_{\text{final,min}}}{R_{C1} \times 6} = \frac{106.40}{0.15 \times 6} \approx 118.22\,\text{N} \tag{5b}$$

4.2.3. Apply Hertzian Contact Pressure Formula to Find Ball Diameter

The Hertzian contact pressure $p_{\max}$ for a spherical contact can be approximated by:

$$p_{\max} = \frac{F}{\pi a^2} \tag{6a}$$

$$a = \sqrt[3]{\frac{3FR}{4E'}} \tag{6b}$$

$$p_{\max} = \frac{F}{\pi {\frac{3FR}{4E'}}^{\frac{2}{3}}} \tag{6c}$$

Rearrange for d:

$${\frac{3FR}{4E'}}^{\frac{2}{3}} = \frac{F}{p_{\max}\pi} \tag{7a}$$

$$\frac{3FR}{4E'} = {\frac{F}{p_{\max}\pi}}^{\frac{3}{2}} \tag{7b}$$

$$R = \frac{4E'}{3F} \cdot {\frac{F}{p_{\max}\pi}}^{\frac{3}{2}} \tag{7c}$$

$$2R = d = 2 \cdot \frac{4E'}{3F} \cdot {\frac{F}{p_{\max}\pi}}^{\frac{3}{2}} \tag{7d}$$

For $p_{\min} = 1.2\,\text{GPa}$ and for $p_{\max} = 1.5\,\text{GPa}$:

$$d_{\max} = \frac{8 \times 220 \cdot 10^9}{3 \times 118.22} \cdot {\frac{118.22}{1.2 \cdot 10^9 \cdot \pi}}^{\frac{3}{2}} = 27.55\text{mm} \tag{8a}$$

$$d_{\min} = \frac{8 \times 220 \cdot 10^9}{3 \times 167.34} \cdot {\frac{167.34}{1.5 \cdot 10^9 \cdot \pi}}^{\frac{3}{2}} = 23.46\text{mm} \tag{8b}$$

To keep the Hertzian contact pressure between 1.2 and 1.5 GPa, the diameter of each ball should be in the range:

$$d = 23.56\,\text{mm to}\,27.55\,\text{mm}$$

5. Gears

5.1. Type of Gears

Spur or helical gears would be suitable. Both are used for parallel-shaft configurations where constant speed ratios are required. Spur Gears are easier to manufacture and maintain, but noisier, especially at higher speeds. Helical Gears have a quieter operation and smoother engagement due to angled teeth, ideal for high-speed applications but slightly less efficient due to sliding friction

Helical gears are preferred here due to their quieter operation and smoother power transmission, which can be beneficial in a machine tool environment, especially at 1000 RPM.

5.2. Number of Teeth

For the fixed-stage gears, we need to establish a ratio of 0.698. Assume this stage consists of two steps on parallel shafts. The Matlab code will be in the appendix. In this code there are limits on the maximum and minimum number of teeth a gear can have and then multiple nested for loops to find the closest teeth number to the correct ratio, in this case 0.698.

For this code the minimum was 10 and the maximum was 200.

In this case the code has given these values:
- Pair 1: Driver = 149, Driven = 10
- Pair 2: Driver = 10, Driven = 104

Quick check to ensure that the numbers are correct:

$$\frac{10}{149} \times \frac{104}{10} = 0.6980 \tag{9}$$

Getting 0.6980 means that the number of teeth are correct. This also means that the middle gear has the same number of teeth that is can be got rid of entirely. Which means greater efficiency.

6. Power Calculations

6.1. Power and Torque

Output power, efficiencies and known gear ratios will be used to back-calculate input requirements. Then, based on these requirements, a suitable power element that matches the torque and power specifications for the application will be selected.

6.1.1. Effective Transmission Ratio

$$i_{\text{total,max}} = i_{\max} \times \text{Fixed Gear Ratio} \tag{10a}$$

$$0.750 \times 0.698 = 0.5235 \tag{10b}$$

$$i_{\text{total,min}} = i_{\min} \times \text{Fixed Gear Ratio} \tag{10c}$$

$$0.525 \times 0.698 = 0.3665 \tag{10d}$$

This gives a total range of transmission ratios from 0.3665 to 0.5235.

6.1.2. Efficiencies

For the fixed state gear box's helical gears:
- Efficiency per gear pair: 97% (typical for helical gears)(2).
- Total Efficiency: $0.97^2 = 94.09\%$.

So, assuming no loss in the shaft or connections, the fixed state gear box has an efficiency of 94.09

Estimating CVT efficiency using traction coefficient:

$$0.99^6 = 0.941 \tag{11}$$

Each contact has about 99% efficiency. However there are six balls, which means six contacts which make the total efficiency for the CVT 0.941.

There are six bearings within the fixed gear stage aswell, two for each shaft. Each bearing is a radial

Material	Characteristics	Surface Hardness	Core Toughness	Typical Applications
Alloy Steel (e.g., AISI 8620)	High strength, good wear resistance	High (after carburizing)	High	High-load applications, industrial gears
Carbon Steel (e.g., AISI 1045)	Moderate strength, lower wear resistance than alloy steel	Moderate (if hardened)	Moderate	Low-load gears, smaller mechanical systems
Nitrided Steel (e.g., AISI 4340 nitrided)	Excellent wear resistance due to nitriding	Very High	Moderate to High	High-wear applications, helical and bevel gears
Cast Iron	Good wear resistance, vibration damping, but lower toughness	Moderate	Low	Low-speed gears, applications where weight and damping are important

Table 2: Characteristics of Gear Materials

Failure Mode	Description	Causes	Prevention Methods
Pitting and Spalling	Formation of pits or flakes on the gear tooth surface due to surface fatigue from cyclic loading	High contact stress, inadequate lubrication, surface imperfections	Use hardened materials (e.g., carburized or nitrided steels), high-quality lubricants, and smooth surface finishes
Tooth Bending Fatigue	Cracks at the tooth root due to repeated bending stresses, potentially leading to fracture	Cyclic bending stress, insufficient core toughness	Select materials with high core toughness, such as carburized alloy steels, and optimize tooth profile design to reduce stress
Scuffing (Adhesive Wear)	Surface damage from adhesive wear due to insufficient lubrication or excessive sliding between teeth	Poor lubrication, high sliding speeds, excessive contact pressure	Use carburized steel with a smooth surface finish, ensure proper lubrication, reduce sliding forces
Microcracking	Formation of small cracks on the gear surface due to high localized stresses, potentially leading to spalling	High Hertzian contact stress, inadequate material toughness	Select high-hardness materials, maintain proper lubrication, reduce stress concentrations
Abrasive Wear	Gradual material loss from contact with abrasive particles or direct metal-to-metal contact	Contaminants in lubricant, insufficient hardness	Regular lubrication maintenance, filtration to remove contaminants, and use of surface-hardened materials

Table 3: Common Failure Modes in Gears and Prevention Methods

ball bearing and has an assumed efficiency of 0.99. Therefore it also has an efficiency of 0.941.

There is also one cylindrical roller bearing in the CVT section of the mechanism, which means there is an assumed efficiency of 0.96 with that.

So,

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta_{\text{gear}} \times \eta_{\text{CVT}}} = \frac{5833}{0.9409 \times 0.941^2 \times 0.96} \approx 7292.86 \text{ W} \tag{12}$$

Using the relationship between power, torque, and speed, torque can be found:

$$P_{\text{in}} = T_{\text{in}} \times \omega_{\text{in}} \tag{13a}$$

$$T_{\text{in}} = \frac{P_{\text{in}}}{\omega_{\text{in}}} \tag{13b}$$

$$\frac{7292.86 \text{ W}}{104.72 \text{ rad/s}} = 69.64 \text{ Nm} \tag{13c}$$

6.2. Power Element

The motor must meet the following requirements:

- Power: At least 7292.86 W
- Speed: 1000 RPM.
- Torque: At least 69.6 Nm.

6.2.1. Motor Type

A 3-phase AC induction motor or a brushless DC motor might be good for this application, as they are capable of providing the required power and torque.

3-Phase AC induction motors are widely available and reliable. They are suitable for industrial applications and can handle continuous operation. BLDC motors are more efficient than induction motors and provides precise control over speed and torque.

6.2.2. Final Motor Choice

The EMRAX 207 more than fulfills previous requirements set out by this paper. It has power up to 40 kW which is surplus to requirements. It operates efficiently at lower RPM ranges and offers continuous torque well above 69.6 Nm. This model comfortably meets the requirements.

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7. Materials and Failures

7.1. Gear Materials

Characteristics of different gear materials are shown in table 2.

Considering power (7.3 kW), speed (1000 RPM), and torque (69.64 Nm), a carburized alloy steel, like AISI 8620 for the gear teeth, would likely be the best option for durability and strength, particularly for high-load, high-torque conditions.

7.2. Potential Failure modes

Several failure modes can occur in gear systems. The most relevant are shown in table 3.

8. Conclusion

The two-stage transmission design, with a CVT and fixed gear, is efficient, and durable. Using radial and cylindrical bearings and carburized alloy steel gears, it minimizes failure risks while maximizing performance. Load distribution, Hertzian pressure, and efficiency calculations are done.

REFERENCES

[1] Garcia, L., *Design of Mechanisms and Machines* CVT Transmission project, University of Sussex, 2024

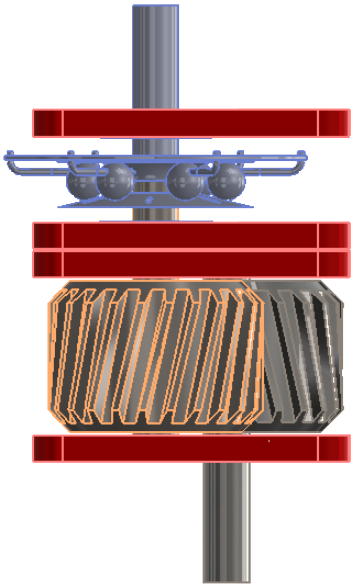
[2] Budynas, R.G. and Nisbett, J.K., 2020. *Shigley's Mechanical Engineering Design.* 11th ed. New York: McGraw-Hill Education.

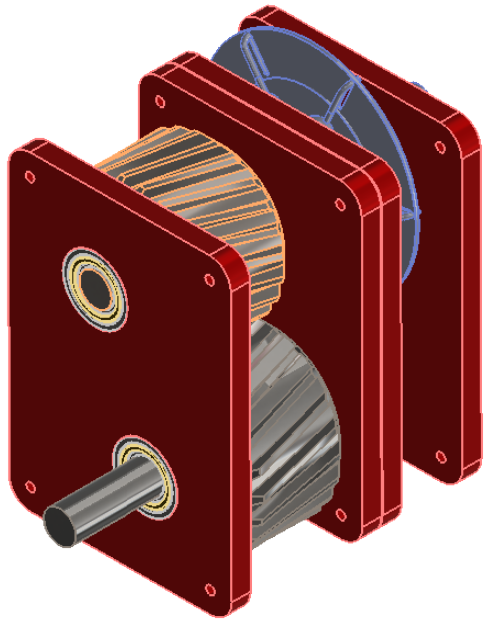
```
close all
clear
clc

% Target total gear ratio
target_ratio = 0.336;

% Tolerance range for acceptable ratio deviation
tolerance = 0.001; % or adjust as necessary

% Range of possible teeth counts for driver and driven gears
min_teeth = 10;  % Minimum number of teeth (adjust based on practical limits)
max_teeth = 200; % Maximum number of teeth (adjust as necessary)

% Initialize variables to store the best match
best_ratio = Inf;
best_teeth = [];

% Loop through all possible combinations of teeth counts
for N_driver1 = min_teeth:max_teeth
    for N_driven1 = min_teeth:max_teeth
        for N_driver2 = min_teeth:max_teeth
            for N_driven2 = min_teeth:max_teeth

                % Calculate individual gear ratios for each stage
                ratio1 = N_driven1 / N_driver1;
                ratio2 = N_driven2 / N_driver2;

                % Calculate the total gear ratio
                total_ratio = ratio1 * ratio2;

                % Check if this ratio is closer to the target
                if abs(total_ratio - target_ratio) < abs(best_ratio -
target_ratio)
                    best_ratio = total_ratio;
                    best_teeth = [N_driver1, N_driven1, N_driver2,
N_driven2];
                end

                % Stop search if within tolerance
                if abs(total_ratio - target_ratio) < tolerance
                    break;
                end
            end
            if abs(best_ratio - target_ratio) < tolerance
                break;
            end
        end
        if abs(best_ratio - target_ratio) < tolerance
            break;
        end
    end
```

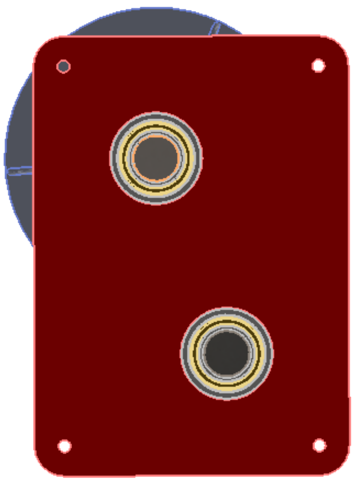
```
end

% Display the best combination found
fprintf('Best ratio: %.4f\n', best_ratio);
fprintf('Optimal teeth counts:\n');
fprintf(' - Pair 1: Driver = %d, Driven = %d\n', best_teeth(1),
best_teeth(2));
fprintf(' - Pair 2: Driver = %d, Driven = %d\n', best_teeth(3),
best_teeth(4));
```

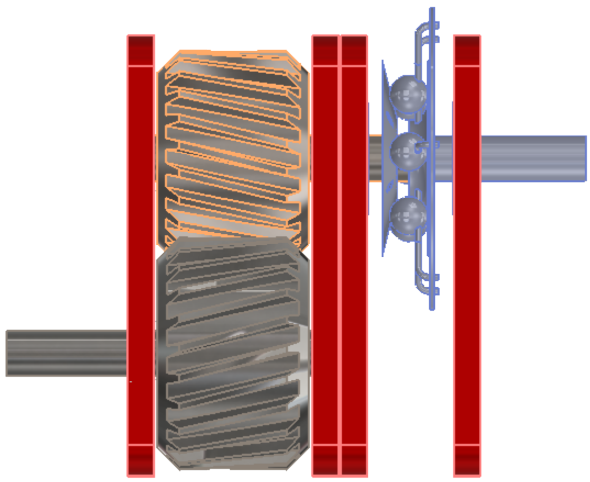
```
Best ratio: 0.3360
Optimal teeth counts:
 - Pair 1: Driver = 125, Driven = 10
 - Pair 2: Driver = 10, Driven = 42
```

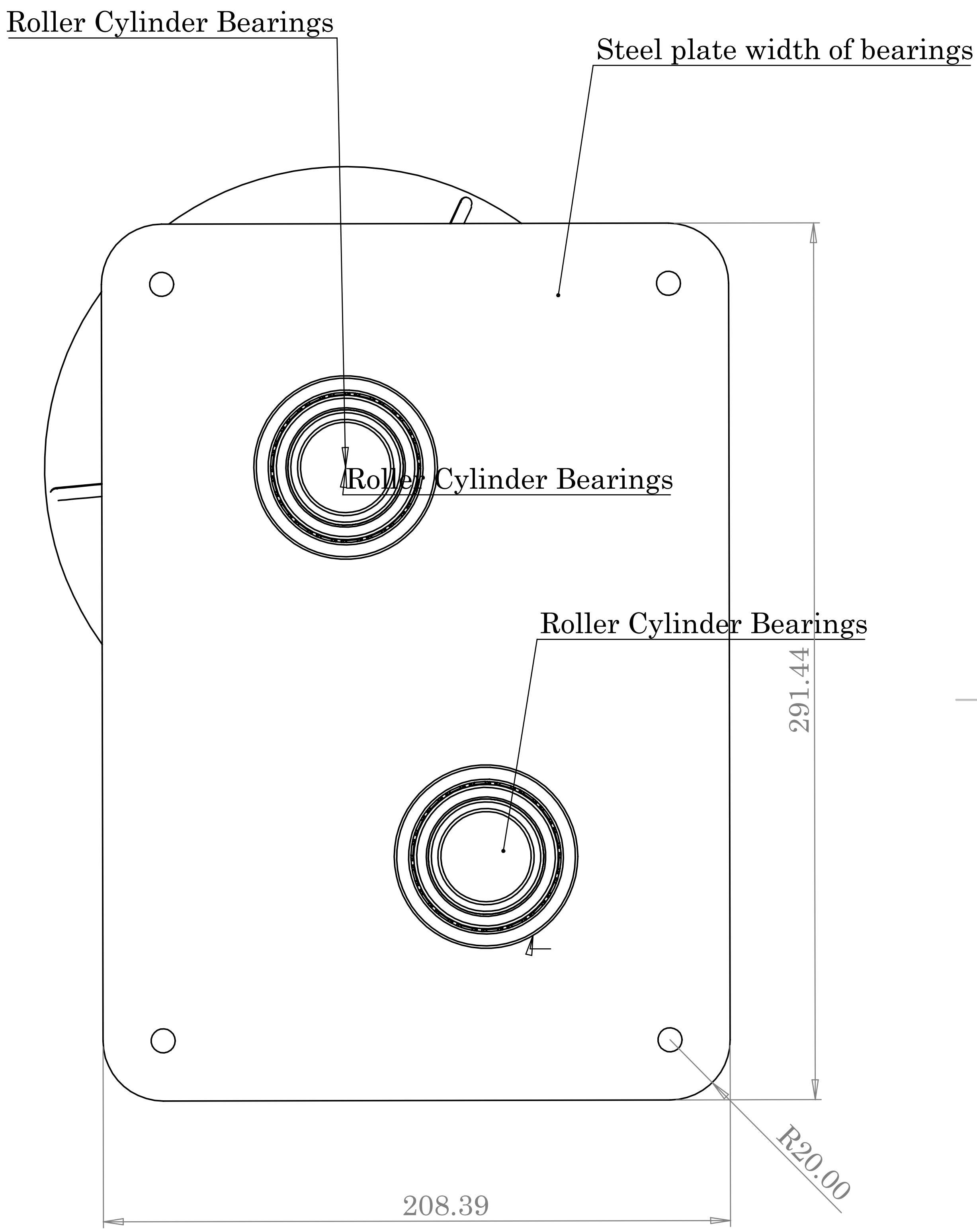
Published with MATLAB® R2024a

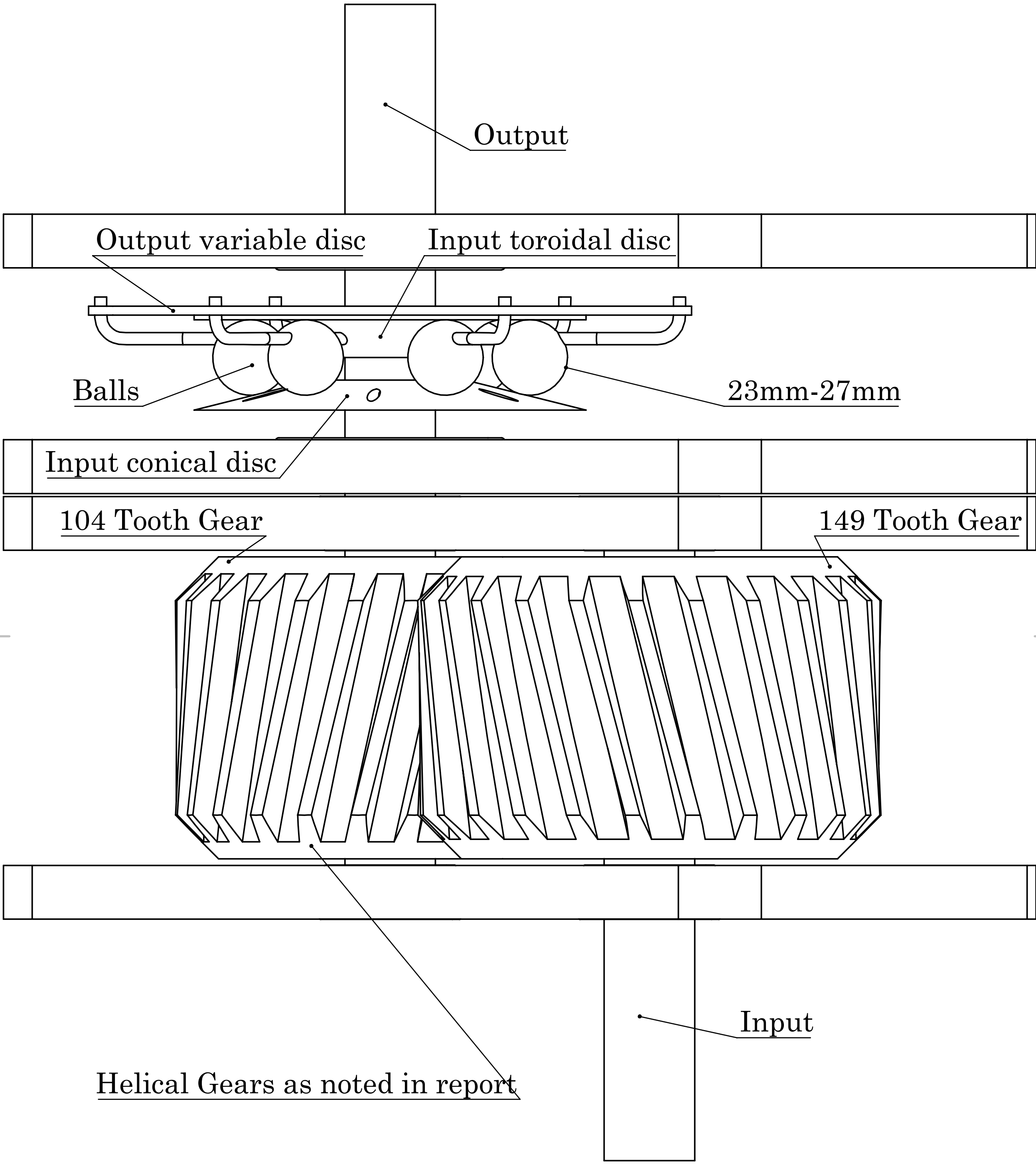









Roller Cylinder Bearings
Steel plate width of bearings
Roller Cylinder Bearings
Roller Cylinder Bearings
291.44
R20.00
208.39


Output
Output variable disc
Input toroidal disc
Balls
23mm-27mm
Input conical disc
104 Tooth Gear
149 Tooth Gear
Input
Helical Gears as noted in report